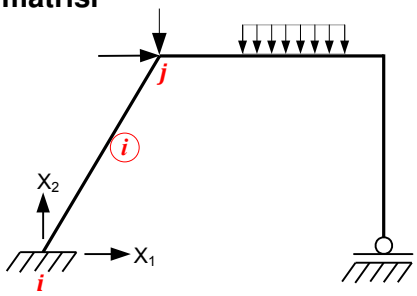
9. Düzlem çerçeve elemanı rijitlik ve transformasyon matrisi

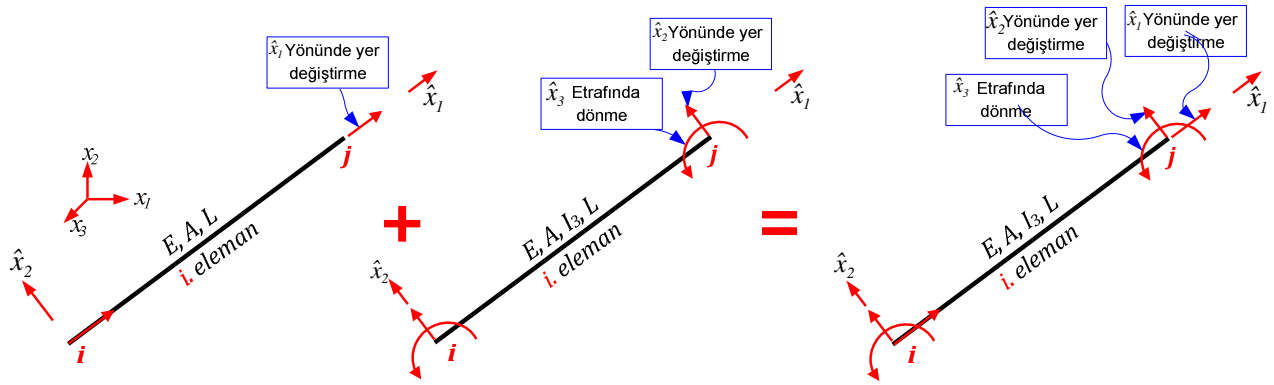
Sağdaki, x_1-x_2 düzleminde olan çerçevenin çözümünde kullanılabilecek *i.* elemanının rijitlik ve transformasyon matrisi belirlenecektir. Yükler çerçeve düzlemindedir, burulma yoktur. Çerçevenin herhangi bir noktasında serbestlik derecesi=3 tür: Bir düşey+bir yatay yer değiştirme+x_3 etrafında dönme.



Şekil 9.1 Düzlem çerçeve

9.1 Yerel rijitlik matrisi

Normal kuvvet sadece elemanın ekseni boyunca yer değiştirmeye(uzama-kısalma) neden olur, kesme ve momenti etkilemez. Bu nedenle çerçeve elemanı kafes eleman ile kiriş elemanın toplamı olarak düşünebiliriz:



Düzlem kafes eleman | Düzlem kiriş eleman | Düzlem çerçeve eleman

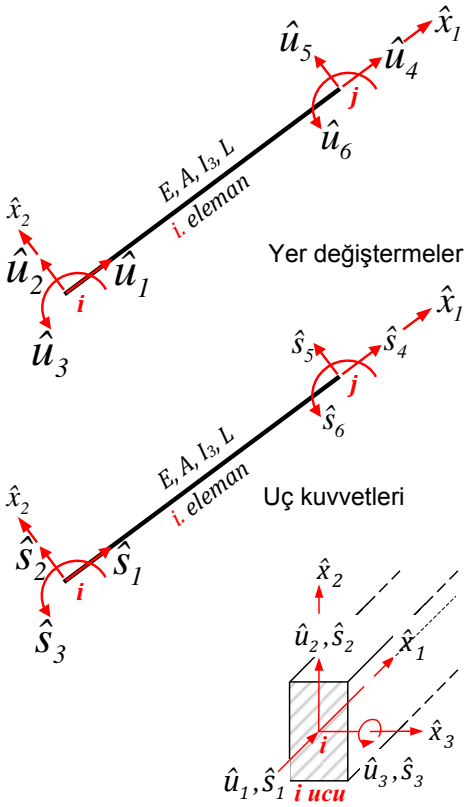
Çerçeve elemanın yerel koordinatlardaki yer değiştirmeleri ve kuvvetleri şekil 9.2 de gösterilmiştir. $\hat{x}_3$ lokal ekseni kâğıt(ekran) düzlemine dik ve okuyucuya doğru yönlenmiştir. Yerel yer değiştirme ve kuvvet vektörleri:

$$\underline{\hat{u}}^i = \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \\ \hat{u}_4 \\ \hat{u}_5 \\ \hat{u}_6 \end{bmatrix} \begin{matrix} i \\ \\ \\ j \\ \\ \end{matrix}$$

$\hat{u}_1$: *i* noktasında $\hat{x}_1$ yönünde yer değiştirme
$\hat{u}_2$: *i* noktasında $\hat{x}_2$ yönünde yer değiştirme
$\hat{u}_3$: *i* noktasında $\hat{x}_3$ etrafında dönme
$\hat{u}_4$: *j* noktasında $\hat{x}_1$ yönünde yer değiştirme
$\hat{u}_5$: *j* noktasında $\hat{x}_2$ yönünde yer değiştirme
$\hat{u}_6$: *j* noktasında $\hat{x}_3$ etrafında dönme

$$\underline{\hat{s}}^i = \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \\ \hat{s}_4 \\ \hat{s}_5 \\ \hat{s}_6 \end{bmatrix} \begin{matrix} i \\ \\ \\ j \\ \\ \end{matrix}$$

$\hat{s}_1$: *i* noktasında $\hat{x}_1$ yönünde eksenel kuvvet
$\hat{s}_2$: *i* noktasında $\hat{x}_2$ yönünde kesme kuvveti
$\hat{s}_3$: *i* noktasında $\hat{x}_3$ etrafında moment
$\hat{s}_4$: *j* noktasında $\hat{x}_1$ yönünde eksenel kuvvet
$\hat{s}_5$: *j* noktasında $\hat{x}_2$ yönünde kesme kuvveti
$\hat{s}_6$: *j* noktasında $\hat{x}_3$ etrafında moment



Şekil 9.2: Düzlem çerçeve eleman

5.12 deki kafes elemanın 2·2 boyutlu rijitlik matrisini ve 8.14 deki kiriş elemanın rijitlik matrisini 6·6 boyutluya dönüştürelim

Bak: 5.12

$$\underline{\hat{k}}^i_{kafes} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \rightarrow \underline{\hat{k}}^i_{kafes} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Bak: 8.14

$$\underline{\hat{k}}^i_{kiriş} = \begin{bmatrix} \frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} & -\frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} \\ \frac{6EI_3}{L^2} & \frac{4EI_3}{L} & -\frac{6EI_3}{L^2} & \frac{2EI_3}{L} \\ -\frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} & \frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} \\ \frac{6EI_3}{L^2} & \frac{2EI_3}{L} & -\frac{6EI_3}{L^2} & \frac{4EI_3}{L} \end{bmatrix} \rightarrow \underline{\hat{k}}^i_{kiriş} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} & 0 & -\frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{4EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{2EI_3}{L} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} & 0 & \frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{2EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{4EI_3}{L} \end{bmatrix}$$

Düzlem çerçeve elemanın 6·6 boyutlu rıjitlik matrisi $\underline{\hat{k}}^i_{çerçeve} = \underline{\hat{k}}^i_{kafes} + \underline{\hat{k}}^i_{kiriş}$ dir:

$$\underline{\hat{k}}^i_{çerçeve} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} & 0 & -\frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{4EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{2EI_3}{L} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} & 0 & \frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{2EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{4EI_3}{L} \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} & 0 & -\frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{4EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{2EI_3}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} & 0 & \frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{2EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{4EI_3}{L} \end{bmatrix}$$

$$\underline{\hat{k}}^i = \begin{array}{c} \\ i \\ \\ \\ j \\ \\ \\ \end{array} \overset{\begin{matrix} \quad i & & & \quad j & & \end{matrix}}{\left[\begin{array}{ccc|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} & 0 & -\frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{4EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{2EI_3}{L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} & 0 & \frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{2EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{4EI_3}{L} \end{array}\right]}$$

Çerçeve elemanın yerel rijitlik matrisi

(9.1)

9.2 Düzlem çerçeve elemanın yerel denge koşulu

8.18 bağıntısı geçerlidir. Sadece matrislerin boyut ve içeriği farklıdır. $\underline{\tilde{s}}^i$ eleman üzerindeki yüklerin eşdeğeri(ankastrelik kuvvetleri) olmak üzere:

$$\underline{\hat{k}}^i\,\underline{\hat{u}}^i + \underline{\tilde{s}}^i = \underline{\hat{s}}^i \tag{9.2}$$

$$\underbrace{\begin{array}{c} \\ i \\ \\ \\ j \\ \\ \\ \end{array} \overset{\begin{matrix} \quad i & & & \quad j & & \end{matrix}}{\left[\begin{array}{ccc|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} & 0 & -\frac{12EI_3}{L^3} & \frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{4EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{2EI_3}{L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} & 0 & \frac{12EI_3}{L^3} & -\frac{6EI_3}{L^2} \\ 0 & \frac{6EI_3}{L^2} & \frac{2EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & \frac{4EI_3}{L} \end{array}\right]}}_{\underline{\hat{k}}^i} \underbrace{\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \\ \hat{u}_4 \\ \hat{u}_5 \\ \hat{u}_6 \end{bmatrix}}_{\underline{\hat{u}}^i} + \underbrace{\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \end{bmatrix}}_{\underline{\tilde{s}}^i} = \underbrace{\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \\ \hat{s}_4 \\ \hat{s}_5 \\ \hat{s}_6 \end{bmatrix}}_{\underline{\hat{s}}^i} \tag{9.3}$$

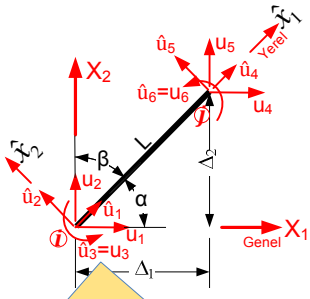
Eleman üzerindeki yüklerin eşdeğer yükleri

Eleman yerel kuvvetleri

Eleman yerel yer değiştirmeleri

Eleman yerel rijitlik matrisi

9.2 Düzlem çerçeve elemanın transformasyon matrisi



Dönme ve moment, eksenlerin dönmesinden etkilenmez. Çünkü x_3 ve $\hat{x}_3$ eksenleri daima birbirine paraleldir.

x_{1i}, x_{2i} : i noktasının genel koordinatları
x_{1j}, x_{2j} : j noktasının genel koordinatları

$$\Delta_1 = x_{1j} - x_{1i},\quad \Delta_2 = x_{2j} - x_{2i},\quad L = \sqrt{{\Delta_1}^2 + {\Delta_2}^2},\quad Cos\,\alpha = \frac{\Delta_1}{L}, Cos\,\beta = \frac{\Delta_2}{L}$$

$$\begin{aligned}
\hat{u}_1 &= u_1\frac{\Delta_1}{L}, +u_2\frac{\Delta_2}{L}\\
\hat{u}_2 &= -u_1\frac{\Delta_2}{L}, +u_2\frac{\Delta_1}{L}\\
\hat{u}_3 &= u_3\\
\hat{u}_4 &= u_4\frac{\Delta_1}{L}, +u_5\frac{\Delta_2}{L}\\
\hat{u}_5 &= -u_4\frac{\Delta_2}{L}, +u_5\frac{\Delta_1}{L}\\
\hat{u}_6 &= u_6
\end{aligned}
\qquad
\underline{T}^i = \begin{array}{c} \\ i \\ \\ \\ j \\ \\ \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} i & & & & j & \end{array} \\ \left[\begin{array}{cccccc}
\Delta_1/L & \Delta_2/L & 0 & 0 & 0 & 0\\
-\Delta_2/L & \Delta_1/L & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & \Delta_1/L & \Delta_2/L & 0\\
0 & 0 & 0 & -\Delta_2/L & \Delta_1/L & 0\\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] \end{array} \qquad (9.4)$$

Eleman yerel yer değiştirmeleri — Eleman transformasyon matrisi — Eleman genel yer değiştirmeleri

$$\underbrace{\hat{\underline{u}}^i}_{\hat{\underline{u}}^i} = \underbrace{\underline{T}^i}_{\underline{T}^i}\underbrace{\underline{u}^i}_{\underline{u}^i} = \begin{bmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\\ \hat{u}_4\\ \hat{u}_5\\ \hat{u}_6\end{bmatrix} = \begin{array}{c} \\ i \\ \\ \\ j \\ \\ \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} i & & & & j & \end{array} \\ \left[\begin{array}{cccccc}
\Delta_1/L & \Delta_2/L & 0 & 0 & 0 & 0\\
-\Delta_2/L & \Delta_1/L & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & \Delta_1/L & \Delta_2/L & 0\\
0 & 0 & 0 & -\Delta_2/L & \Delta_1/L & 0\\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] \end{array} \begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\end{bmatrix} \qquad (9.5)$$

Eleman genel eşdeğer kuvvetleri — Eleman transformasyon matrisinin transpozu — Eleman yerel eşdeğer kuvvetleri

$$\underbrace{\bar{\underline{s}}^i}_{\bar{\underline{s}}^i} = \underbrace{(\underline{T}^i)^T}_{(\underline{T}^i)^T}\underbrace{\tilde{\underline{s}}^i}_{\tilde{\underline{s}}^i} = \begin{bmatrix}\bar{s}_1\\ \bar{s}_2\\ \bar{s}_3\\ \bar{s}_4\\ \bar{s}_5\\ \bar{s}_6\end{bmatrix} = \begin{array}{c} \\ i \\ \\ \\ j \\ \\ \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} i & & & & j & \end{array} \\ \left[\begin{array}{cccccc}
\Delta_1/L & -\Delta_2/L & 0 & 0 & 0 & 0\\
\Delta_2/L & \Delta_1/L & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & \Delta_1/L & -\Delta_2/L & 0\\
0 & 0 & 0 & \Delta_2/L & \Delta_1/L & 0\\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] \end{array} \begin{bmatrix}\tilde{s}_1\\ \tilde{s}_2\\ \tilde{s}_3\\ \tilde{s}_4\\ \tilde{s}_5\\ \tilde{s}_6\end{bmatrix} \qquad (9.6)$$

9.3 Düzlem çerçeve elemanın genel rijitlik matrisi

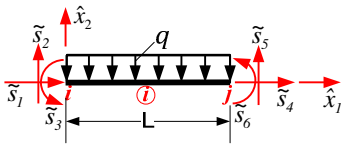
$$\underline{k}^i = (\underline{T}^i)^T\hat{\underline{k}}^i\underline{T}^i \qquad (9.7)$$

$$\underline{k}^i = \begin{array}{c} \\ i \\ \\ \\ j \\ \\ \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} i & & & & j & \end{array} \\ \left[\begin{array}{cccccc}
\frac{EA}{L^3}\Delta_1^2 + 12\frac{EI_3}{L^5}\Delta_2^2 & & & & & \\
(\frac{EA}{L^3} - 12\frac{EI_3}{L^5})\Delta_1\Delta_2 & \frac{EA}{L^3}\Delta_2^2 + 12\frac{EI_3}{L^5}\Delta_1^2 & & & Simetrik & \\
-6\frac{EI_3}{L^3}\Delta_2 & 6\frac{EI_3}{L^3}\Delta_1 & 4\frac{EI_3}{L} & & & \\
-(\frac{EA}{L^3}\Delta_1^2 + 12\frac{EI_3}{L^5}\Delta_2^2) & -(\frac{EA}{L^3} - 12\frac{EI_3}{L^5})\Delta_1\Delta_2 & 6\frac{EI_3}{L^3}\Delta_2 & \frac{EA}{L^3}\Delta_1^2 + 12\frac{EI_3}{L^5}\Delta_2^2 & & \\
-(\frac{EA}{L^3} - 12\frac{EI_3}{L^5})\Delta_1\Delta_2 & -(\frac{EA}{L^3}\Delta_2^2 + 12\frac{EI_3}{L^5}\Delta_1^2) & -6\frac{EI_3}{L^3}\Delta_1 & (\frac{EA}{L^3} - 12\frac{EI_3}{L^5})\Delta_1\Delta_2 & \frac{EA}{L^3}\Delta_2^2 + 12\frac{EI_3}{L^5}\Delta_1^2 & \\
-6\frac{EI_3}{L^3}\Delta_2 & 6\frac{EI_3}{L^3}\Delta_1 & 2\frac{EI_3}{L} & 6\frac{EI_3}{L^3}\Delta_2 & -6\frac{EI_3}{L^3}\Delta_1 & 4\frac{EI_3}{L}
\end{array}\right] \end{array} \qquad (9.8)$$

9.4 Yüklü elemanın eşdeğer düğüm yükleri

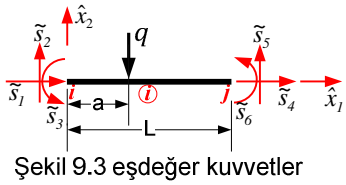
Eleman üzerindeki dış yüklerin enerji eşdeğerleri sistem düğümlerine aktarılır. Tekil ve düzgün yayılı yükün eşdeğer yükleri(ankastrelik kuvvetleri) aşağıda verilmiştir. Diğer yük tiplerinin eşdeğer yükleri EK3 den veya yapı statiği kitaplarından alınabilir.

$$\underbrace{\tilde{s}^i}_{\tilde{s}^i} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{qL}{2} \\ \frac{qL^2}{12} \\ 0 \\ \frac{qL}{2} \\ -\frac{qL^2}{12} \end{bmatrix}$$



Şekil 9.2 eşdeğer kuvvetler

$$\underbrace{\tilde{s}^i}_{\tilde{s}^i} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 1-\frac{3a^2}{L^2}+\frac{2a^3}{L^3} \\ a-\frac{2a^2}{L}+\frac{a^3}{L^2} \\ 0 \\ \frac{3a^2}{L^2}-\frac{2a^3}{L^3} \\ -\frac{a^2}{L}+\frac{a^3}{L^2} \end{bmatrix} q$$

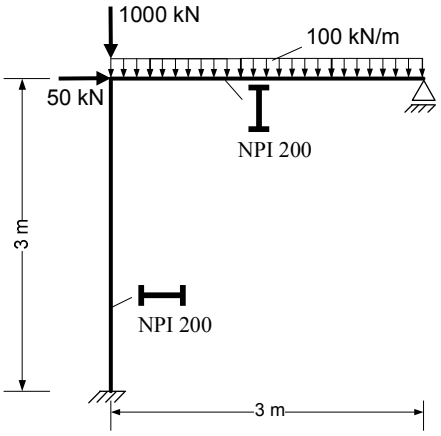


Şekil 9.3 eşdeğer kuvvetler

Eşdeğer yükler yerel koordinatlarda verilmiştir. Eleman yatay, düşey veya eğimli olabileceğinden, sistemin düğüm noktalarına aktarılmadan önce 9.6 bağıntısı ile genel koordinat eksenlerine dönüştürülmesi gerekir.

Sayısal örnek 9.1:

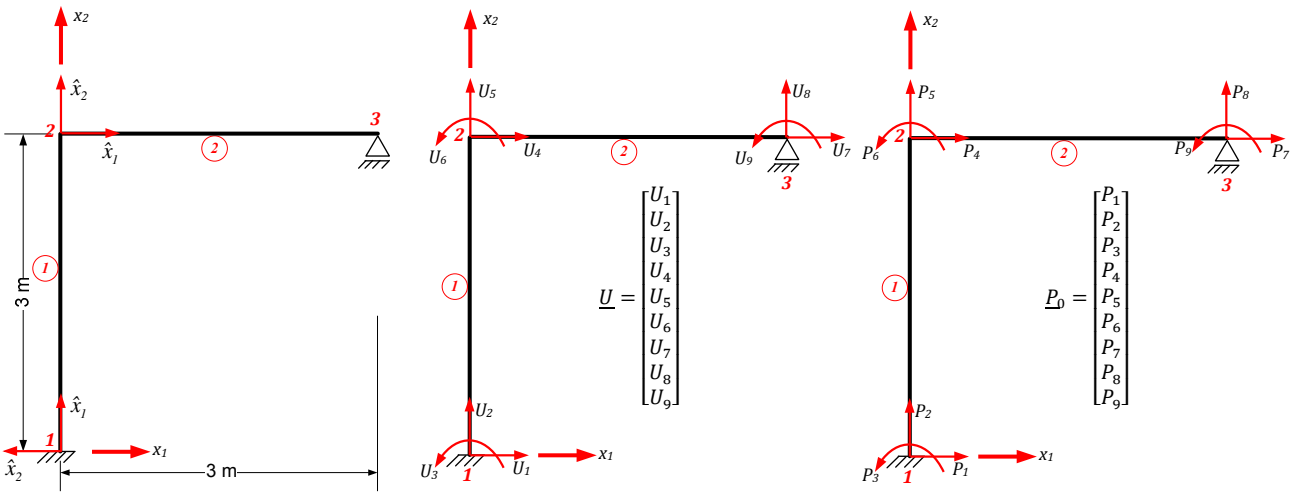
Sağda görülen çerçeve sistem NPI 200 çelik profili ile inşa edilecektir. Kesme, moment ve normal kuvvet diyagramlarını çiziniz.



HAZIRLIK:
Sistem 2 elemanla modellenmiştir. Seçilen koordinat sistemleri, numaralandırma, düğüm yer değiştirmelerinin ve yüklerin adları şekil 9.4 de verilmiştir. 2 nolu elemanın yayılı yük eşdeğeri sistem düğümlerine aktarılacaktır.

Hesaplarda kN ve m birimlerini kullanalım. NPI 200 için elastisite modülü: $E=2.1\cdot10^5$ N/mm^2 (yapı çeliği için)$=21\cdot10^7$ kN/m^2, Kesit alanı: A=33.4 cm^2=$334\cdot10^{-5}$ m^2, Atalet momenti: I_3=2140 cm^4=$214\cdot10^{-7}$ m^4 (çelik profil tablosundan)

Düğüm serbestlik derecesi(bir yatay, bir düşey yer değiştirme, bir dönme)= 3, Sistem serbestlik derecesi=3·3=9



Koordinat sistemleri ve numaralandırma | Sistem yer değiştirmeleri | Sistem düğüm yükleri

Şekil 9.4: Sistemin modellenmesi

El hesaplarını kolaylaştırmak için aşağıdaki çizelgeyi hazırlamak yaralıdır. Birimler kN ve m dir.

Eleman	*E*	*A*	*I_3*	*i ucu*	*j ucu*	*Koordinatlar*		*Δ_1*	*Δ_2*	*L*	*EA/L*	*EI_3/L*	*$\frac{\Delta_1}{L}$*	*$\frac{\Delta_2}{L}$*
						i	*j*							
1	$21\cdot10^7$	$334\cdot10^{-5}$	$214\cdot10^{-7}$	1	2	0, 0	0, 3	0	3	3	233800	1498	0	1
2	$21\cdot10^7$	$334\cdot10^{-5}$	$214\cdot10^{-7}$	2	3	0, 3	3, 3	3	0	3	233800	1498	1	0

9. Düzlem çerçeve elemanı rijitlik ve transformasyon matrisi

Elemanların transformasyon matrisleri: Bak 9.4

$$\underline{T}^1=\begin{bmatrix}0&1&0&0&0&0\\-1&0&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&1&0\\0&0&0&-1&0&0\\0&0&0&0&0&1\end{bmatrix},\quad \underline{T}^2=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

Elemanların yerel rijitlik matrisleri: Bak 9.1

$$\underline{\hat{k}}^1=\begin{array}{c}\\1\\2\end{array}\begin{array}{c}\begin{array}{ccc}\color{red}{1}&&\end{array}\\\left[\begin{array}{ccc|ccc}233800&0&0&-233800&0&0\\0&1997&2996&0&-1997&2996\\0&2996&5992&0&-2996&2996\\\hline-233800&0&0&233800&0&0\\0&-1997&-2996&0&1997&-2996\\0&2996&2996&0&-2996&5992\end{array}\right]\end{array}\begin{array}{c}\\\\\color{red}{2}\end{array}$$

$$\underline{\hat{k}}^2=\begin{array}{c}\\2\\3\end{array}\begin{array}{c}\begin{array}{ccc}\color{red}{2}&&\end{array}\\\left[\begin{array}{ccc|ccc}233800&0&0&-233800&0&0\\0&1997&2996&0&-1997&2996\\0&2996&5992&0&-2996&2996\\\hline-233800&0&0&233800&0&0\\0&-1997&-2996&0&1997&-2996\\&2996&2996&0&-2996&5992\end{array}\right]\end{array}\begin{array}{c}\\\\\color{red}{3}\end{array}$$

Elemanların genel rijitlik matrisleri ve sistem rijitlik matrisi: $\underline{k}^i=(\underline{T}^i)^T\underline{\hat{k}}^i\underline{T}^i$

$$\underline{k}^1=\underbrace{\begin{bmatrix}0&-1&0&0&0&0\\1&0&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&-1&0\\0&0&0&1&0&0\\0&0&0&&0&1\end{bmatrix}}_{(\underline{T}^1)^T}\underbrace{\begin{bmatrix}233800&0&0&-233800&0&0\\0&1997&2996&0&-1997&2996\\0&2996&5992&0&-2996&2996\\-233800&0&0&233800&0&0\\0&-1997&-2996&0&1997&-2996\\0&2996&2996&0&-2996&5992\end{bmatrix}}_{\underline{\hat{k}}^1}\underbrace{\begin{bmatrix}0&1&0&0&0&0\\-1&0&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&1&0\\0&0&0&-1&0&0\\0&0&0&0&0&1\end{bmatrix}}_{\underline{T}^1}$$

$$\underline{k}^1=\begin{array}{c}\\1\\2\end{array}\begin{array}{c}\begin{array}{ccc}\color{red}{1}&&\end{array}\\\left[\begin{array}{ccc|ccc}1997&0&-2996&-1997&0&-2996\\0&233800&0&0&-233800&0\\-2996&0&5992&2996&0&2996\\\hline-1997&0&2996&1997&0&2996\\0&-233800&0&0&233800&0\\-2996&0&2996&2996&0&5992\end{array}\right]\end{array}\begin{array}{c}\\\\\color{red}{2}\end{array}$$

$$\underline{k}^2=(\underline{T}^2)^T\underline{\hat{k}}^2\underline{T}^2=\begin{array}{c}\\2\\3\end{array}\begin{array}{c}\begin{array}{ccc}\color{red}{2}&&\end{array}\\\left[\begin{array}{ccc|ccc}233800&0&0&-233800&0&0\\0&1997&2996&0&-1997&2996\\0&2996&5992&0&-2996&2996\\\hline-233800&0&0&233800&0&0\\0&-1997&-2996&0&1997&-2996\\0&2996&2996&0&-2996&5992\end{array}\right]\end{array}\begin{array}{c}\\\\\color{red}{3}\end{array}$$

$\underline{T}^2$ birim matris olduğundan çarpımı yapmaya gerek yoktur, $\underline{k}^2=\underline{\hat{k}}^2$ dir.

$$\underline{K}_0=\begin{array}{c}\\1\\2\\3\end{array}\begin{array}{c}\begin{array}{ccc}\color{red}{1}&&\end{array}\\\left[\begin{array}{ccc|ccc|ccc}1997&0&-2996&-1997&0&-2996&0&0&0\\0&233800&0&0&-233800&0&0&0&0\\-2996&0&5992&2996&0&2996&0&0&0\\\hline-1997&0&2996&235797&0&2996&-233800&0&0\\0&-233800&0&0&235797&2996&0&-1997&2996\\-2996&0&2996&2996&2996&11984&0&-2996&2996\\\hline0&0&0&-233800&0&0&233800&0&0\\0&0&0&0&-1997&-2996&0&1997&-2996\\0&0&0&0&2996&2996&0&-2996&5992\end{array}\right]\end{array}\begin{array}{c}\\\\\\\color{red}{2}\\\\\\\color{red}{3}\end{array}$$

Sistem rijitlik matrisi

Eleman eşdeğer yükleri:

1. eleman üzerinde yük yok

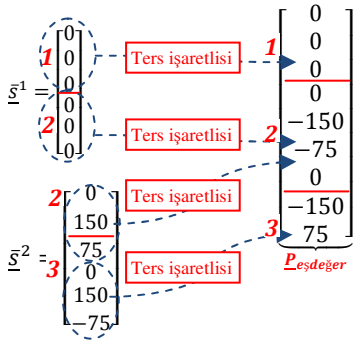
$$\underline{\tilde{s}}^1=\begin{bmatrix}\tilde{s}_1\\\tilde{s}_2\\\tilde{s}_3\\\tilde{s}_4\\\tilde{s}_5\\\tilde{s}_6\end{bmatrix}=\begin{array}{c}\color{red}{1}\\\\\\\color{red}{2}\end{array}\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}\xrightarrow{\text{DÖNÜŞTÜR}}\underline{\bar{s}}^1=(\underline{T}^1)^T\underline{\tilde{s}}^1=\begin{bmatrix}\bar{s}_1\\\bar{s}_2\\\bar{s}_3\\\bar{s}_4\\\bar{s}_5\\\bar{s}_6\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

2. elemanın düzgün yayılı yükünün yerel eşdeğer tekil yükleri. Bak şekil 9.2

Yerel eşdeğer kuvvetlerin genel eşdeğer yüklere dönüştürülmesi, Bak: 9.6. $\underline{T}^2$ birim matris olduğundan $\underline{\bar{s}}^2=\underline{\tilde{s}}^2$ dir.

$$\underline{\tilde{s}}^2=\begin{bmatrix}\tilde{s}_1\\\tilde{s}_2\\\tilde{s}_3\\\tilde{s}_4\\\tilde{s}_5\\\tilde{s}_6\end{bmatrix}=\begin{array}{c}\color{red}{2}\\\\\\\color{red}{3}\end{array}\begin{bmatrix}0\\150\\75\\0\\150\\-75\end{bmatrix}\xrightarrow{\text{DÖNÜŞTÜR}}\underline{\bar{s}}^2=(\underline{T}^2)^T\underline{\tilde{s}}^2=\begin{bmatrix}\bar{s}_1\\\bar{s}_2\\\bar{s}_3\\\bar{s}_4\\\bar{s}_5\\\bar{s}_6\end{bmatrix}=\begin{array}{c}\\\\\color{red}{2}\\\\\\\color{red}{3}\end{array}\begin{bmatrix}0\\150\\75\\0\\150\\-75\end{bmatrix}$$

Sistem yük vektörü:



$$\underline{P}_0 = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ 50 \\ -1000 \\ 0 \\ 0 \\ P_8 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -150 \\ -75 \\ 0 \\ -150 \\ 75 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ 50 \\ -1150 \\ -75 \\ 0 \\ P_8 - 150 \\ 75 \end{bmatrix}$$

Düğümlerde toplam yük. P_1, P_2, P_3, P_8: Reaksiyonlar

Düğümlerde verilmiş yükler ve reaksiyonlar

$\underline{P}_{eşdeğer}$ $\underline{P}_0$

Sistem denge koşulu: $\underline{K}_0 \underline{U} = \underline{P}_0$

$$\begin{bmatrix} 1997 & 0 & -2996 & -1997 & 0 & -2996 & 0 & 0 & 0 \\ 0 & 233800 & 0 & 0 & -233800 & 0 & 0 & 0 & 0 \\ -2996 & 0 & 5992 & 2996 & 0 & 2996 & 0 & 0 & 0 \\ -1997 & 0 & 2996 & 235797 & 0 & 2996 & -233800 & 0 & 0 \\ 0 & -233800 & 0 & 0 & 235797 & 2996 & 0 & -1997 & 2996 \\ -2996 & 0 & 2996 & 2996 & 2996 & 11984 & 0 & -2996 & 2996 \\ 0 & 0 & 0 & -233800 & 0 & 0 & 233800 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1997 & -2996 & 0 & 1997 & -2996 \\ 0 & 0 & 0 & 0 & 2996 & 2996 & 0 & -2996 & 5992 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ 50 \\ -1150 \\ -75 \\ 0 \\ P_8 - 150 \\ 75 \end{bmatrix}$$

$\underline{K}_0$ $\underline{U}$ $\underline{P}_0$

Sınır koşulları ve işlenmesi: $1 \cdot U_1 = 0, \quad 1 \cdot U_2 = 0, \quad 1 \cdot U_3 = 0, \quad 1 \cdot U_8 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 235797 & 0 & 2996 & -233800 & 0 & 0 \\ 0 & 0 & 0 & 0 & 235797 & 2996 & 0 & 0 & 2996 \\ 0 & 0 & 0 & 2996 & 2996 & 11984 & 0 & 0 & 2996 \\ 0 & 0 & 0 & -233800 & 0 & 0 & 233800 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2996 & 2996 & 0 & 0 & 5992 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \\ -1150 \\ -75 \\ 0 \\ 0 \\ 75 \end{bmatrix}$$

$\underline{K}$ $\underline{U}$ $\underline{P}$

ÇÖZÜM

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \\ 0.070163\ m \\ 0\ m \\ 0.029994\ rad \end{bmatrix}$$

$\underline{U}$ $\underline{U}$

Denge kontrolü ve reaksiyonlar: $\underline{P}_{hesap} = \underline{K}_0\ \underline{U} - \underline{P}_{eşdeğer}$

$$\begin{bmatrix} 1997 & 0 & -2996 & -1997 & 0 & -2996 & 0 & 0 & 0 \\ 0 & 233800 & 0 & 0 & -233800 & 0 & 0 & 0 & 0 \\ -2996 & 0 & 5992 & 2996 & 0 & 2996 & 0 & 0 & 0 \\ -1997 & 0 & 2996 & 235797 & 0 & 2996 & -233800 & 0 & 0 \\ 0 & -233800 & 0 & 0 & 235797 & 2996 & 0 & -1997 & 2996 \\ -2996 & 0 & 2996 & 2996 & 2996 & 11984 & 0 & -2996 & 2996 \\ 0 & 0 & 0 & -233800 & 0 & 0 & 233800 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1997 & -2996 & 0 & 1997 & -2996 \\ 0 & 0 & 0 & 0 & 2996 & 2996 & 0 & -2996 & 5992 \end{bmatrix} \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \\ 0.070163\ m \\ 0\ m \\ 0.029994\ rad \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -150 \\ -75 \\ 0 \\ -150 \\ 75 \end{bmatrix} = \begin{bmatrix} -50\ kN \\ 1140\ kN \\ 120\ kNm \\ 50\ kN \\ -1000\ kN \\ 0\ kNm \\ 0\ kNm \\ 150\ kN \\ 0\ kNm \end{bmatrix}$$

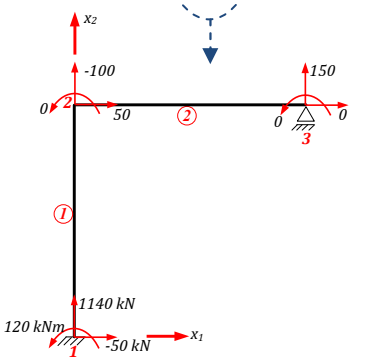
$\underline{K}_0$ $\underline{U}$ $\underline{P}_{eşdeğer}$ $\underline{P}_{hesap}$

Reaksiyon, Reaksiyon, Reaksiyon, ✓, ✓, ✓, ✓, Reaksiyon, ✓

Elemanların genel yer değiştirmeleri:

$$\underline{u}^1 = \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \end{bmatrix}, \quad \underline{u}^2 = \begin{bmatrix} 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \\ 0.070163\ m \\ 0\ m \\ 0.029994\ rad \end{bmatrix}$$

$\underline{u}^1$ $\underline{u}^2$



Hesaplanan reaksiyon ve düğüm kuvvetleri

Elemanların yerel yer değiştirmeleri: $\underline{\hat{u}}^i = \underline{T}^i \underline{u}^i$ Bak 9.5

$$\underline{\hat{u}}^1 = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{T}^1} \underbrace{\begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \end{bmatrix}}_{\underline{u}^1} = \begin{matrix} 1 \\ \\ \\ \\ 2 \\ \\ \end{matrix} \underbrace{\begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ -0.004876\ m \\ -0.070163\ m \\ -0.030079\ rad \end{bmatrix}}_{\underline{\hat{u}}^1}$$

$$\underline{\hat{u}}^2 = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{T}^2} \underbrace{\begin{bmatrix} 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \\ 0.070163\ m \\ 0\ m \\ 0.029994\ rad \end{bmatrix}}_{\underline{u}^2} = \begin{matrix} 2 \\ \\ \\ 3 \\ \\ \\ \end{matrix} \underbrace{\begin{bmatrix} 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \\ 0.070163\ m \\ 0\ m \\ 0.029994\ rad \end{bmatrix}}_{\underline{\hat{u}}^2}$$

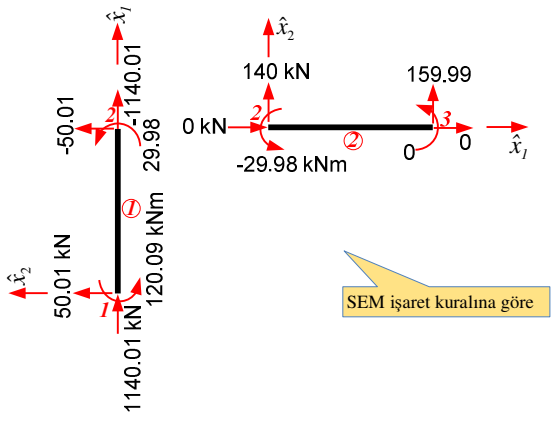
Elemanların yerel kuvvetleri: $\underline{\hat{k}}^i\ \underline{\hat{u}}^i + \underline{\tilde{s}}^i = \underline{\hat{s}}^i$ Bak 9.2

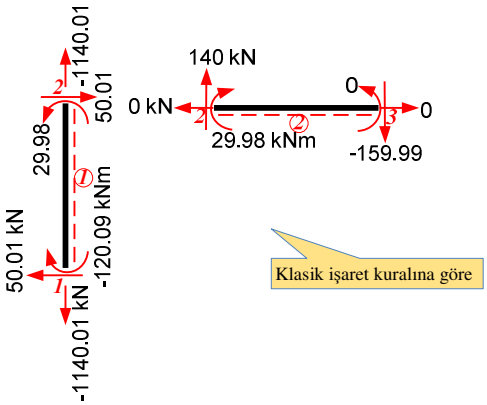
Eleman eşdeğer kuvvetleri

$$\underbrace{\begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \\ \hat{S}_4 \\ \hat{S}_5 \\ \hat{S}_6 \end{bmatrix}}_{\underline{\hat{s}}^1} = \underbrace{\begin{bmatrix} 233800 & 0 & 0 & -233800 & 0 & 0 \\ 0 & 1997 & 2996 & 0 & -1997 & 2996 \\ 0 & 2996 & 5992 & 0 & -2996 & 2996 \\ -233800 & 0 & 0 & 233800 & 0 & 0 \\ 0 & -1997 & -2996 & 0 & 1997 & -2996 \\ 0 & 2996 & 2996 & 0 & -2996 & 5992 \end{bmatrix}}_{\underline{\hat{k}}^1} \underbrace{\begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ -0.004876\ m \\ -0.070163\ m \\ -0.030079\ rad \end{bmatrix}}_{\underline{\hat{u}}^1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{\tilde{s}}^1} = \begin{matrix} 1 \\ \\ \\ \\ 2 \\ \\ \end{matrix} \underbrace{\begin{bmatrix} 1140.01\ kN \\ 50.01\ kN \\ 120.09\ kNm \\ -1140.01\ kN \\ -50.01\ kN \\ 29.98\ kNm \end{bmatrix}}_{\underline{\hat{s}}^1}$$

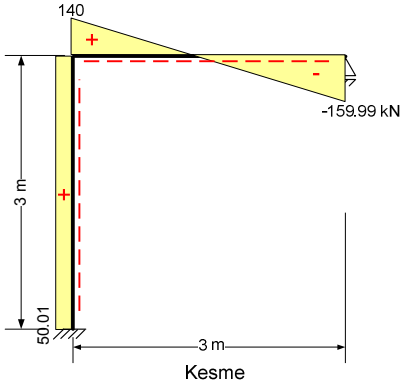
Eleman eşdeğer kuvvetleri

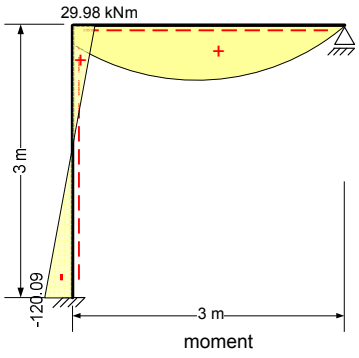
$$\underbrace{\begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \\ \hat{S}_4 \\ \hat{S}_5 \\ \hat{S}_6 \end{bmatrix}}_{\underline{\hat{s}}^2} = \underbrace{\begin{bmatrix} 233800 & 0 & 0 & -233800 & 0 & 0 \\ 0 & 1997 & 2996 & 0 & -1997 & 2996 \\ 0 & 2996 & 5992 & 0 & -2996 & 2996 \\ -233800 & 0 & 0 & 233800 & 0 & 0 \\ 0 & -1997 & -2996 & 0 & 1997 & -2996 \\ 0 & 2996 & 2996 & 0 & -2996 & 5992 \end{bmatrix}}_{\underline{\hat{k}}^2} \underbrace{\begin{bmatrix} 0.070163\ m \\ -0.004876\ m \\ -0.030079\ rad \\ 0.070163\ m \\ 0\ m \\ 0.029994\ rad \end{bmatrix}}_{\underline{\hat{u}}^2} + \underbrace{\begin{bmatrix} 0 \\ 150 \\ 75 \\ 0 \\ 150 \\ -75 \end{bmatrix}}_{\underline{\tilde{s}}^1} = \begin{matrix} 2 \\ \\ \\ 3 \\ \\ \\ \end{matrix} \underbrace{\begin{bmatrix} 0\ kN \\ 140\ kN \\ -29.98\ kNm \\ 0\ kN \\ 159.99\ kN \\ 0\ kNm \end{bmatrix}}_{\underline{\hat{s}}^2}$$

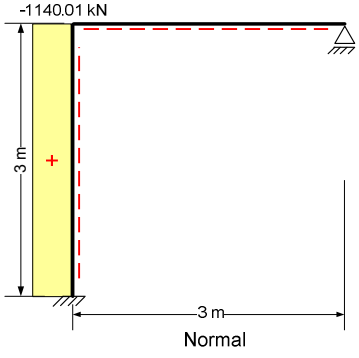




Diyagramlar(klasik işaret kuralına göre):

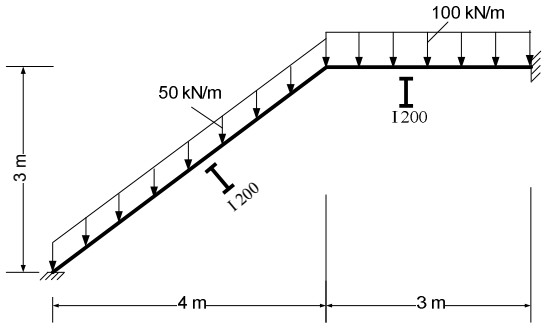






Sayısal örnek 9.2

Sağda görülen çerçeve sistem I 200 profili ile inşa edilecektir. Kesme, moment ve normal kuvvet diyagramlarını çiziniz.

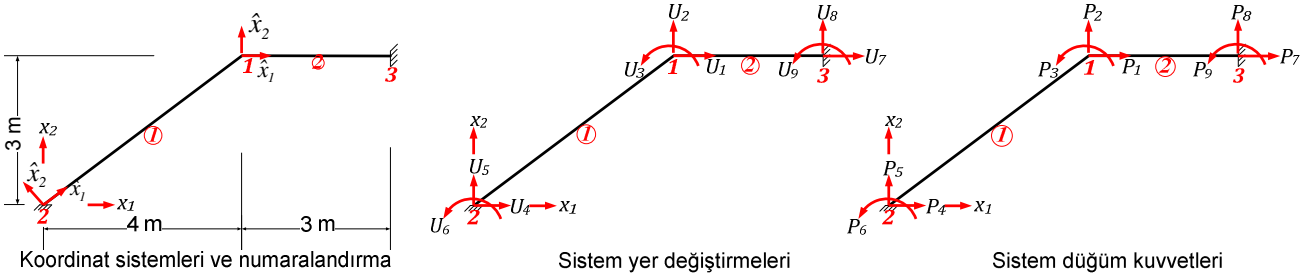


Hesaplanması istenen çerçeve sistem

HAZIRLIK:
Sistem için seçilen koordinat sistemleri, numaralandırma, düğüm yer değiştirmelerinin ve yüklerin adları şekil 9.5 de verilmiştir. Sistem 2 elemanla modellenmiştir. Elemanların yayılı yük eşdeğeri sistem düğümlerine aktarılacaktır.

Hesaplarda kN ve m birimlerini kullanalım.
Elastisite modülü: $E=2.1\cdot10^5$ N/mm^2 (yapı çeliği çin)$=21\cdot10^7$ kN/m^2
Kesit alanı $A=33.4$ cm$^2=334\cdot10^{-5}$ m^2, Atalet momenti: $I_3=2140$ cm$^2=214\cdot10^{-7}$ m^4 (çelik profil tablosundan)

Düğüm serbestlik derecesi(bir yatay, bir düşey yer değiştirme, bir dönme)= 3, Sistem serbestlik derecesi=3·3=9



Şekil 9.5: Sistemin modellenmesi

El hesaplarını için aşağıdaki çizelgeyi hazırlayalım. Birimler kN ve m dir.

Eleman	*E*	*A*	I_3	*i ucu*	*j ucu*	*Koordinatlar*		Δ_1	Δ_2	*L*	*EA/L*	EI_3/L	$\frac{\Delta_1}{L}$	$\frac{\Delta_2}{L}$
						i	*j*							
1	$21\cdot10^7$	$334\cdot10^{-5}$	$214\cdot10^{-7}$	2	1	0, 0	4, 3	4	3	5	140280	898.8	0.8	0.6
2	$21\cdot10^7$	$334\cdot10^{-5}$	$214\cdot10^{-7}$	1	3	4, 3	7, 3	3	0	3	233800	1498	1	0

Elemanların transformasyon matrisleri: Bak 9.4

$$\underline{T}^1=\begin{bmatrix} 0.8 & 0.6 & 0 & 0 & 0 & 0\\ -0.6 & 0.8 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.8 & 0.6 & 0\\ 0 & 0 & 0 & -0.6 & 0.8 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\quad \underline{T}^2=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elemanların yerel rijitlik matrisleri: Bak 9.1

$$\underline{\hat{k}}^1=\begin{array}{c} \\ 2\\ \\ 1\\ \\ \end{array}\!\!\begin{bmatrix} 140280 & 0 & 0 & -140280 & 0 & 0\\ 0 & 431 & 1079 & 0 & -431 & 1079\\ 0 & 1079 & 3595 & 0 & -1079 & 1798\\ -140280 & 0 & 0 & 140280 & 0 & 0\\ 0 & -431 & -1079 & 0 & 431 & -1079\\ 0 & 1079 & 1798 & 0 & -1079 & 3595 \end{bmatrix}\begin{array}{c} \\ 2\\ \\ 1\\ \\ \end{array}$$

$$\underline{\hat{k}}^2=\begin{array}{c} \\ 1\\ \\ 3\\ \\ \end{array}\!\!\begin{bmatrix} 233800 & 0 & 0 & -233800 & 0 & 0\\ 0 & 1997 & 2996 & 0 & -1997 & 2996\\ 0 & 2996 & 5992 & 0 & -2996 & 2996\\ -233800 & 0 & 0 & 233800 & 0 & 0\\ 0 & -1997 & -2996 & 0 & 1997 & -2996\\ 0 & 2996 & 2996 & 0 & -2996 & 5992 \end{bmatrix}\begin{array}{c} \\ 1\\ \\ 3\\ \\ \end{array}$$

Elemanların genel rijitlik matrisleri ve sistem rijitlik matrisi: $\underline{k}^i = (\underline{T}^i)^T \hat{\underline{k}}^i \underline{T}^i$

$$\underline{k}^1 = \underbrace{\begin{bmatrix} 0.8 & -0.6 & 0 & 0 & 0 & 0 \\ 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & -0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{(\underline{T}^1)^T} \underbrace{\begin{bmatrix} 140280 & 0 & 0 & -140280 & 0 & 0 \\ 0 & 431 & 1079 & 0 & -431 & 1079 \\ 0 & 1079 & 3595 & 0 & -1079 & 1798 \\ -140280 & 0 & 0 & 140280 & 0 & 0 \\ 0 & -431 & -1079 & 0 & 431 & -1079 \\ 0 & 1079 & 1798 & 0 & -1079 & 3595 \end{bmatrix}}_{\hat{\underline{k}}^1} \underbrace{\begin{bmatrix} 0.8 & 0.6 & 0 & 0 & 0 & 0 \\ -0.6 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0 \\ 0 & 0 & 0 & -0.6 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{T}^1}$$

$$\underline{k}^1 = \begin{array}{c} \\ 2 \\ \\ \\ 1 \\ \\ \end{array} \begin{bmatrix} \overset{2}{89935} & 67127 & -647 & \overset{1}{-89935} & -67127 & -647 \\ 67127 & 50777 & 863 & -67127 & -50777 & 863 \\ -647 & 863 & 3595 & 647 & -863 & 1798 \\ -89935 & -67127 & 647 & 89935 & 67127 & 647 \\ -67127 & -50777 & -863 & 67127 & 50777 & -863 \\ -647 & 863 & 1798 & 647 & -863 & 3595 \end{bmatrix}$$

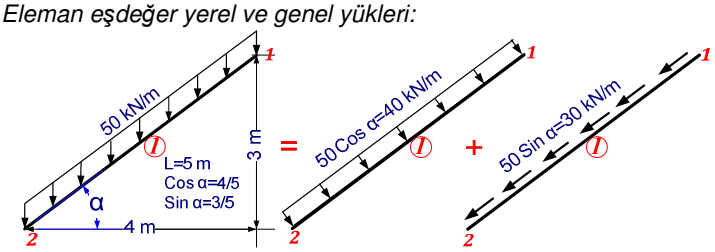
$$\underline{k}^2 = (\underline{T}^2)^T \hat{\underline{k}}^2 \underline{T}^2 = \begin{array}{c} \\ 1 \\ \\ \\ 3 \\ \\ \end{array} \begin{bmatrix} \overset{1}{233800} & 0 & 0 & \overset{3}{-233800} & 0 & 0 \\ 0 & 1997 & 2996 & 0 & -1997 & 2996 \\ 0 & 2996 & 5992 & 0 & -2996 & 2996 \\ -233800 & 0 & 0 & 233800 & 0 & 0 \\ 0 & -1997 & -2996 & 0 & 1997 & -2996 \\ 0 & 2996 & 2996 & 0 & -2996 & 5992 \end{bmatrix}$$

$\underline{T}^2$ birim matris olduğundan çarpımı yapmaya gerek yoktur, $\underline{k}^2 = \hat{\underline{k}}^2$ dir.

$$\underline{K}_0 = \begin{array}{c} \\ 1 \\ \\ \\ 2 \\ \\ \\ 3 \\ \\ \end{array} \begin{bmatrix} \overset{1}{323735} & 67127 & 647 & \overset{2}{-89935} & -67127 & 647 & \overset{3}{-233800} & 0 & 0 \\ 67127 & 52774 & 2133 & -67127 & -50777 & -863 & 0 & -1997 & 2996 \\ 647 & 2133 & 9587 & -647 & 863 & 1798 & 0 & -2996 & 2996 \\ -89935 & -67127 & -647 & 89935 & 67127 & -647 & 0 & 0 & 0 \\ -67127 & -50777 & 863 & 67127 & 50777 & 863 & 0 & 0 & 0 \\ 647 & -863 & 1798 & -647 & 863 & 3595 & 0 & 0 & 0 \\ -233000 & 0 & 0 & 0 & 0 & 0 & 233800 & 0 & 0 \\ 0 & -1997 & -2996 & 0 & 0 & 0 & 0 & 1997 & -2996 \\ 0 & 2996 & 2996 & 0 & 0 & 0 & 0 & -2996 & 5992 \end{bmatrix}$$

Sistem rijitlik matrisi

Eleman eşdeğer yerel ve genel yükleri:



1. elemanın düzgün yayılı yükünün yerel eşdeğer tekil yükleri. Bak: EK3

$$\underline{\tilde{s}}^1 = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 83.33 \\ 0 \\ 100 \\ -83.33 \end{bmatrix} + \begin{bmatrix} 75 \\ 0 \\ 0 \\ 75 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} 2 \\ \\ \\ 1 \\ \\ \end{array} \begin{bmatrix} 75 \\ 100 \\ 83.33 \\ 75 \\ 100 \\ -83.33 \end{bmatrix}$$

$$\underline{\tilde{s}}^1 = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \end{bmatrix} = \begin{array}{c} \\ 2 \\ \\ \\ 1 \\ \\ \end{array} \begin{bmatrix} 75 \\ 100 \\ 83.33 \\ 75 \\ 100 \\ -83.33 \end{bmatrix}$$

DÖNÜŞTÜR

Bak: 9.6

$$\underline{\bar{s}}^1 = (\underline{T}^1)^T \underline{\tilde{s}}^1 = \begin{bmatrix} \bar{s}_1 \\ \bar{s}_2 \\ \bar{s}_3 \\ \bar{s}_4 \\ \bar{s}_5 \\ \bar{s}_6 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.8 & -0.6 & 0 & 0 & 0 & 0 \\ 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & -0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{(\underline{T}^1)^T} \underbrace{\begin{bmatrix} 75 \\ 100 \\ 83.33 \\ 75 \\ 100 \\ -83.33 \end{bmatrix}}_{\underline{\tilde{s}}^1} = \begin{array}{c} 2 \\ \\ \\ 1 \\ \\ \end{array} \underbrace{\begin{bmatrix} 0 \\ 125 \\ 83.33 \\ 0 \\ 125 \\ -83.33 \end{bmatrix}}_{\underline{\bar{s}}^1}$$

2. elemanın düzgün yayılı yükünün yerel eşdeğer tekil yükleri. Bak şekil 9.2

$$\underline{\tilde{s}}^2 = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \end{bmatrix} = \begin{array}{c} \\ 1 \\ \\ \\ 3 \\ \\ \end{array} \begin{bmatrix} 0 \\ 150 \\ 75 \\ 0 \\ 150 \\ -75 \end{bmatrix}$$

DÖNÜŞTÜR

$$\underline{\bar{s}}^2 = (\underline{T}^2)^T \underline{\tilde{s}}^2 = \begin{bmatrix} \bar{s}_1 \\ \bar{s}_2 \\ \bar{s}_3 \\ \bar{s}_4 \\ \bar{s}_5 \\ \bar{s}_6 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{(\underline{T}^2)^T} \underbrace{\begin{bmatrix} 0 \\ 150 \\ 75 \\ 0 \\ 150 \\ -75 \end{bmatrix}}_{\underline{\tilde{s}}^2} = \begin{array}{c} 1 \\ \\ \\ 3 \\ \\ \end{array} \underbrace{\begin{bmatrix} 0 \\ 150 \\ 75 \\ 0 \\ 150 \\ -75 \end{bmatrix}}_{\underline{\bar{s}}^2}$$

Sistem yük vektörü:

$$\underline{\bar{s}}^1 = \begin{bmatrix} 0 \\ 125 \\ 83.33 \\ 0 \\ 125 \\ -83.33 \end{bmatrix}_{\underline{\bar{s}}^1} \quad \underline{\bar{s}}^2 = \begin{bmatrix} 0 \\ 150 \\ 75 \\ 0 \\ 150 \\ -75 \end{bmatrix}_{\underline{\bar{s}}^2}$$

Ters işaretlisi

$$\underbrace{\begin{bmatrix} 0-0 \\ -125-150 \\ 83.33-75 \\ 0 \\ -125 \\ -83.33 \\ 0 \\ -150 \\ 75 \end{bmatrix}}_{\underline{P}_{eşdeğer}} = \underbrace{\begin{bmatrix} 0 \\ -275 \\ 8.33 \\ 0 \\ -125 \\ -83.33 \\ 0 \\ -150 \\ 75 \end{bmatrix}}_{\underline{P}_{eşdeğer}}$$

Düğümlerde toplam yük. $P_4, P_5, P_6, P_7, P_8, P_9$: Reaksiyonlar

$$\underline{P}_0 = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -275 \\ 8.33 \\ 0 \\ -125 \\ -83.33 \\ 0 \\ -150 \\ 75 \end{bmatrix}}_{\underline{P}_{eşdeğer}} = \underbrace{\begin{bmatrix} 0 \\ -275 \\ 8.33 \\ P_4 \\ P_5 - 125 \\ P_6 - 83.33 \\ P_7 \\ P_8 - 150 \\ P_9 + 75 \end{bmatrix}}_{\underline{P}_0}$$

Düğümlerde verilmiş yükler ve reaksiyonlar

9. Düzlem çerçeve elemanı rijitlik ve transformasyon matrisi

Sistem denge koşulu: $\underline{K_0}\,\underline{U} = \underline{P_0}$

$$\begin{bmatrix} 323735 & 67127 & 647 & -89935 & -67127 & 647 & -233800 & 0 & 0 \\ 67127 & 52774 & 2133 & -67127 & -50777 & -863 & 0 & -1997 & 2996 \\ 647 & 2133 & 9587 & -647 & 863 & 1798 & 0 & -2996 & 2996 \\ -89935 & -67127 & -647 & 89935 & 67127 & -647 & 0 & 0 & 0 \\ -67127 & -50777 & 863 & 67127 & 50777 & 863 & 0 & 0 & 0 \\ 647 & -863 & 1798 & -647 & 863 & 3595 & 0 & 0 & 0 \\ -233000 & 0 & 0 & 0 & 0 & 0 & 233800 & 0 & 0 \\ 0 & -1997 & -2996 & 0 & 0 & 0 & 0 & 1997 & -2996 \\ 0 & 2996 & 2996 & 0 & 0 & 0 & 0 & -2996 & 5992 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} 0 \\ -275 \\ 8.33 \\ P_4 \\ P_5 - 125 \\ P_6 - 83.33 \\ P_7 \\ P_8 - 150 \\ P_9 + 75 \end{bmatrix}$$

$\underline{K_0}$ $\quad$ $\underline{U}$ $\quad$ $\underline{P_0}$

Sınır koşulları ve işlenmesi: $1 \cdot U_3 = 0,\quad 1 \cdot U_4 = 0,\quad 1 \cdot U_5 = 0,\quad 1 \cdot U_6 = 0,\quad 1 \cdot U_7 = 0,\quad 1 \cdot U_8 = 0$

$$\begin{bmatrix} 323735 & 67127 & 647 & 0 & 0 & 0 & 0 & 0 & 0 \\ 67127 & 52774 & 2133 & 0 & 0 & 0 & 0 & 0 & 0 \\ 647 & 2133 & 9587 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} 0 \\ -275 \\ 8.33 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{ÇÖZÜM} \quad \Rightarrow \quad \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} 0.001488\ m \\ -0.007200\ m \\ 0.002370\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \end{bmatrix}$$

$\underline{K}$ $\quad$ $\underline{U}$ $\quad$ $\underline{P}$ $\quad$ $\underline{U}$ $\quad$ $\underline{U}$

Denge kontrolü ve reaksiyonlar: $\underline{P}_{hesap} = \underline{K}_0\,\underline{U} - \underline{P}_{eşdeğer}$

$$\begin{bmatrix} 323735 & 67127 & 647 & -89935 & -67127 & 647 & -233800 & 0 & 0 \\ 67127 & 52774 & 2133 & -67127 & -50777 & -863 & 0 & -1997 & 2996 \\ 647 & 2133 & 9587 & -647 & 863 & 1798 & 0 & -2996 & 2996 \\ -89935 & -67127 & -647 & 89935 & 67127 & -647 & 0 & 0 & 0 \\ -67127 & -50777 & 863 & 67127 & 50777 & 863 & 0 & 0 & 0 \\ 647 & -863 & 1798 & -647 & 863 & 3595 & 0 & 0 & 0 \\ -233000 & 0 & 0 & 0 & 0 & 0 & 233800 & 0 & 0 \\ 0 & -1997 & -2996 & 0 & 0 & 0 & 0 & 1997 & -2996 \\ 0 & 2996 & 2996 & 0 & 0 & 0 & 0 & -2996 & 5992 \end{bmatrix} \begin{bmatrix} 0.001488\ m \\ -0.007200\ m \\ 0.002370\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \end{bmatrix} - \begin{bmatrix} 0 \\ -275 \\ 8.33 \\ 0 \\ -125 \\ -83.33 \\ 0 \\ -150 \\ 75 \end{bmatrix} = \begin{bmatrix} -0.06\ kN \\ -0.03\ kN \\ 0\ rad \\ 347.96\ kN \\ 392.76\ kN \\ 94.77\ kNm \\ -347.89\ kN \\ 157.28\ kN \\ -89.47\ kNm \end{bmatrix}$$

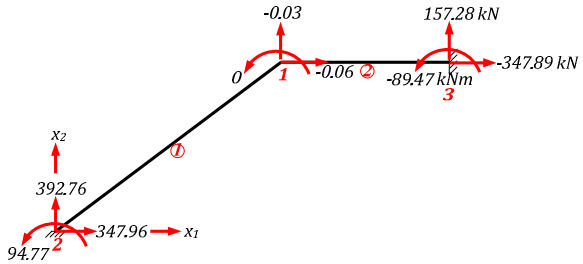
✓ ✓ ✓ Reaksiyon Reaksiyon Reaksiyon Reaksiyon Reaksiyon Reaksiyon

$\underline{K_0}$ $\quad$ $\underline{U}$ $\quad$ $\underline{P}_{eşdeğer}$ $\quad$ $\underline{P}_{hesap}$

Elemanların genel yer değiştirmeleri:

$$\underline{u}^1 = \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ 0.001488\ m \\ -0.007200\ m \\ -0.002370\ rad \end{bmatrix}, \quad \underline{u}^2 = \begin{bmatrix} 0.001488\ m \\ -0.007200\ m \\ -0.002370\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \end{bmatrix}$$

$\underline{u}^1$ $\quad$ $\underline{u}^2$



Hesaplanan reaksiyonlar ve düğüm kuvvetleri

Elemanların yerel yer değiştirmeleri: $\underline{\hat{u}}^i = \underline{T}^i\underline{u}^i$ Bak: 9.5

$$\underline{\hat{u}}^1 = \begin{bmatrix} 0.8 & 0.6 & 0 & 0 & 0 & 0 \\ -0.6 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0 \\ 0 & 0 & 0 & -0.6 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ 0.001488\ m \\ -0.007200\ m \\ -0.002370\ rad \end{bmatrix} = \begin{bmatrix} 0\ m \\ 0\ m \\ 0\ rad \\ -0.003130\ m \\ -0.006653\ m \\ -0.002370\ rad \end{bmatrix}$$

$\underline{T}^1$ $\quad$ $\underline{u}^1$ $\quad$ $\underline{\hat{u}}^1$

$$\underline{\hat{u}}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.001488\ m \\ -0.007200\ m \\ -0.002370\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \end{bmatrix} = \begin{bmatrix} 0.001488\ m \\ -0.007200\ m \\ -0.002370\ rad \\ 0\ m \\ 0\ m \\ 0\ rad \end{bmatrix}$$

$\underline{T}^2$ $\quad$ $\underline{u}^2$ $\quad$ $\underline{\hat{u}}^2$

9. Düzlem çerçeve elemanı rijitlik ve transformasyon matrisi

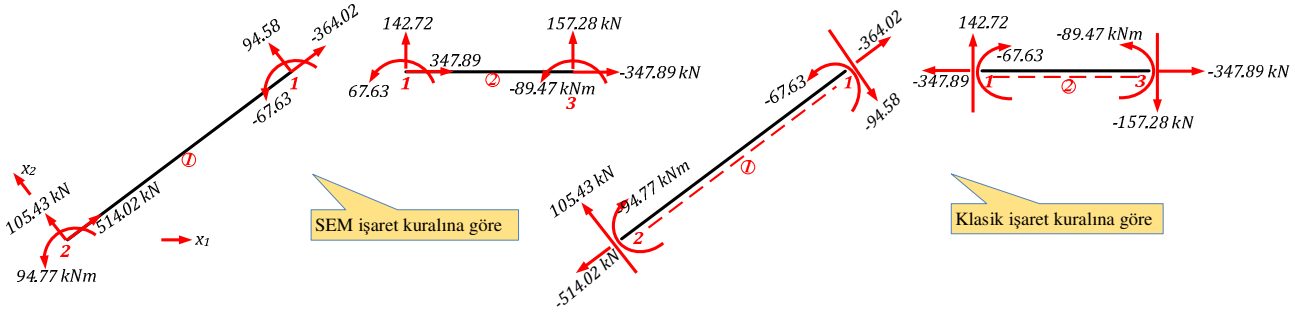
Elemanların yerel kuvvetleri: $\hat{\underline{k}}^i\ \hat{\underline{u}}^i + \tilde{\underline{s}}^i = \hat{\underline{s}}^i$ Bak 9.2

Eleman eşdeğer kuvvetleri

$$\underbrace{\begin{bmatrix}\hat{S}_1\\ \hat{S}_2\\ \hat{S}_3\\ \hat{S}_4\\ \hat{S}_5\\ \hat{S}_6\end{bmatrix}}_{\hat{\underline{s}}^i}{}^1 = \underbrace{\begin{bmatrix}140280 & 0 & 0 & -140280 & 0 & 0\\ 0 & 431 & 1079 & 0 & -431 & 1079\\ 0 & 1079 & 3595 & 0 & -1079 & 1798\\ -140280 & 0 & 0 & 140280 & 0 & 0\\ 0 & -431 & -1079 & 0 & 431 & -1079\\ 0 & 1079 & 1798 & 0 & -1079 & 3595\end{bmatrix}}_{\hat{\underline{k}}^1}\underbrace{\begin{bmatrix}0\\ 0\\ 0\\ -0.003130\ m\\ -0.006653\ m\\ -0.002370\ rad\end{bmatrix}}_{\hat{\underline{u}}^1} + \underbrace{\begin{bmatrix}75\\ 100\\ 83.33\\ 75\\ 100\\ -83.33\end{bmatrix}}_{\tilde{\underline{s}}^1} = \underbrace{\begin{bmatrix}514.02\ kN\\ 105.43\ kN\\ 94.77\ kNm\\ -364.02\ kN\\ 94.58\ kN\\ -67.63\ kNm\end{bmatrix}}_{\hat{\underline{s}}^1}\begin{matrix}2\\ \\ \\ \\ 1\\ \\ \end{matrix}$$

Eleman eşdeğer kuvvetleri

$$\underbrace{\begin{bmatrix}\hat{S}_1\\ \hat{S}_2\\ \hat{S}_3\\ \hat{S}_4\\ \hat{S}_5\\ \hat{S}_6\end{bmatrix}}_{\hat{\underline{s}}^i}{}^2 = \underbrace{\begin{bmatrix}233800 & 0 & 0 & -233800 & 0 & 0\\ 0 & 1997 & 2996 & 0 & -1997 & 2996\\ 0 & 2996 & 5992 & 0 & -2996 & 2996\\ -233800 & 0 & 0 & 233800 & 0 & 0\\ 0 & -1997 & -2996 & 0 & 1997 & -2996\\ 0 & 2996 & 2996 & 0 & -2996 & 5992\end{bmatrix}}_{\hat{\underline{k}}^2}\underbrace{\begin{bmatrix}0.001488\ m\\ -0.007200\ m\\ -0.002370\ rad\\ 0\ m\\ 0\ m\\ 0\ rad\end{bmatrix}}_{\hat{\underline{u}}^2} + \underbrace{\begin{bmatrix}0\\ 150\\ 75\\ 0\\ 150\\ -75\end{bmatrix}}_{\tilde{\underline{s}}^1} = \underbrace{\begin{bmatrix}347.89\ kN\\ 142.72\ kN\\ 67.63\ kNm\\ -347.89\ kN\\ 157.28\ kN\\ -89.47\ kNm\end{bmatrix}}_{\hat{\underline{s}}^2}\begin{matrix}1\\ \\ \\ \\ 3\\ \\ \end{matrix}$$



Diyagramlar(klasik işaret kuralına göre):